

# Comparing Numbers

How do we compare two numbers, say  $a$  and  $b$ ?

$a - b$ or $b - a$	difference
$ a - b $	absolute (or symmetric) difference
$\frac{ a-b }{ a }$	relative (to $a$ ) difference
$\frac{ a-b }{ b }$	relative (to $b$ ) difference
$100 * \frac{ a-b }{ a }$	percent difference ( $a$ )
$r = \frac{b}{a}$	" $b$ is $r$ times $a$ "
$r = 100 * \frac{b}{a}$	" $b$ is $r$ percent of $a$ "
$r = 100 * (\frac{b}{a} - 1)$	" $b$ is $r$ percent more than $a$ "
$r = 100 * (1 - \frac{b}{a})$	" $b$ is $r$ percent less than $a$ "
	and so on...

The first two are absolute comparisons; the rest are relative comparisons. Let's only use the absolute difference ( $|a - b|$ ) and the relative difference ( $|a - b|/|a|$ ). If we are thinking of  $a$  as an approximation to  $b$  we might say that "the absolute error in  $b$  is"  $|a - b|$ . If  $a \neq 0$ , we might say that "the relative error in  $b$  is"  $|a - b|/|a|$  or "the relative error in approximating  $a$  is"  $|a - b|/|a|$ .

You can view absolute difference as comparing numbers in units of 1. Likewise, you can see the relative difference  $|a - b|/|a|$  as comparing  $a$  and  $b$  in  $a$ -units:  $|a - b|/|a| = .23$  means  $|a - b|$  is 0.23  $a$ 's.

As an example, let's look at U.S. Social Security Taxes. All wage earners pay social security (OASDI) taxes at a rate of 12.4 % of the first \$117,000 of income (as of 2014). This is a comparison of the tax you (with your employer) pay to your income in units of your income. For every dollar you earn (up to \$117,000), you (with your employer) pay 12.4 cents (right now, you and your employer each pay 6.2, and the self-employed pay all 12.4 cents). 12.4 cents is the absolute tax on 1 dollar, and 12.4% is the rate.

Suppose you make \$117,000 this year. Then you (with your employer) will pay about \$14,508 in OASDI. Someone who makes \$58,500 will pay about \$7,254, while someone who makes \$1,170,000 will pay about \$14,508. In an absolute sense, you and the person who has 10 times your income pay the same tax, while you pay \$7,254 more than someone who has half your income. In a relative sense you and the person who has half your income pay the same tax rate, while you pay 10 times the tax rate of someone who has 10 times your income. Relative difference 10: your tax rate is 10 times higher than someone who makes 10 times your income. Absolute difference 0: the difference in the amount of tax you and someone with 10 times your income pays is 0.