Computing the Roots of $ax^2 + bx + c$

You might find it silly to devote a page to this computation, after all we have the *quadratic formula*

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Well, let's see. Whether a computation is easy to code (write program(s) to implement) depends on programmer(s), algorithms and data, language(s), target machine(s), time, memory and accuracy requirements, etc.

How will we represent our quadratic polynomial (i.e. what are our *input variables*)? We expect the user to provide a, b and c, and the user needs to know that (a, b, c) corresponds to $ax^2 + bx + c$ and not, e.g. $a + bx + cx^2$. This is a fundamental communication problem, and even if your documentation is impeccable, there is no guarantee the user will understand (or even read) it. At some point we will be satisfied with our documentation, but also know that even if we think we understand each other, stuff happens. For example, what if $a = 0$? We might check that and act accordingly.

How will we communicate the results? Will we print numbers to a computer screen? Plot a figure? Sound tones? If we simply place 2 numbers in memory, then how will the user interpret these? What if the roots are complex? What if something went wrong? We assume that our user is another program (the *calling routine*), so we will not print, plot, blink, vibrate or sound tones (let another routine do that stuff). We will simply place our output in memory as *output variables*. The output and associated documentation must be unambiguous, and we should act accordingly.

Ahh, but this is a floating point computation...

Suppose that our computer language has a data type for real numbers (floats), but not for complex numbers. It is reasonable to assume that the built-in square root function (*sqrt*, say) will give good results for non-negative arguments, but an error for negative arguments. We will need to check the argument $d = b^2 - 4ac$ before calling *sqrt*, and act accordingly.

What does $a = 0$ mean? Is 10^{-24} a small number? Does division by $2a$ give overflow? That depends not only on a, but also b and c. We want to avoid overflow and should act accordingly.

What happens if $b^2 - 4ac \approx b^2$? In that case, we expect (possibly severe) cancellation in exactly one of $-b + \sqrt{d}$ or $-b - \sqrt{d}$. We want to avoid this and should act accordingly. [If $b^2 - 4ac \approx 0$, the cancellation is unavoidable...]

Every piece of code and every computation has a context. There are many different good approaches (and even more bad) to this problem. It is your job (your challenge and your satisfaction) to decide what "act accordingly" means.