Name: _____

(25) 1. Let $A = \begin{bmatrix} -1 & 2 & 0 \\ -3 & 11 & 0 \\ 0 & 10 & 2 \end{bmatrix}$.

(a) Compute the $A = LU$ factorization of $A$ (give $L$ and $U$).

(b) Compute the $PA = LU$ factorization of $A$ (give $L$, $P$, and $U$).

(c) Explain why pivoting is used in Gaussian elimination and give an example of a nonsingular matrix that does not have an $LU$ factorization.

(30) 2. On Conditioning and stability

(a) When is a problem illconditioned?

(b) What is a condition number supposed to tell us?

(c) Define (backward) stability.

(d) Using the ideas of stability and conditioning, describe the circumstances for which the result of a computation will be guaranteed to have high accuracy.

(e) Discuss the backward stability results for solving $Ax = b$ using Gaussian elimination (i) with no pivoting, and (ii) with partial pivoting.

(25) 3. Let $A \in \mathbb{R}^{n \times n}$ be nonsingular. Briefly outline an appropriate method for solving $Ax = b$ in each of the following situations. Include computational costs (in terms of flops). For example, do *not* give details on how we get $L$ or $U$, but give the cost of computing them, and how we use them to find $x$.

(a) Using Gaussian elimination with partial pivoting.

(b) If $A$ is triangular.

(c) If $A$ is symmetric.

(d) If $A$ is symmetric positive definite.

(10) 4. Given the system $Ax = b$, and an approximation $\hat{x}$ to $x$, define the residual $\hat{r} = b - A\hat{x}$. Suppose that $\|\cdot\|$ is submultiplicative. Show that

$$\frac{\|x - \hat{x}\|}{\|x\|} \leq \kappa(A)\frac{\|\hat{r}\|}{\|b\|}$$

(10) 5. Compute the Cholesky factorization of $A = \begin{bmatrix} 4 & -2 \\ -2 & 10 \end{bmatrix}$.