Name: _____

(28) 1. Define $H \equiv H(u) = I - \frac{2}{u^t u} uu^t$.

(a) Show that $H^2 = I$.

(b) If $x \in \mathbb{R}^m$ is nonzero, which vector u should we use so that $Hx = \alpha e_1$?

(c) Given $u \in \mathbb{R}^m$ and $B \in \mathbb{R}^{m \times n}$, how many flops is required to compute HB?

(d) Let $u = (0, 3, 1)^t$ and let $x = (2, 3, 0)^t$. Compute Hx.

(27) 2. Let $A \in \mathbb{R}^{m \times n}$, $m > n$ be full rank.

(a) Describe the Gram-Schmidt QR factorization of A (not the process, but the properties of the output and the cost in flops).

(b) Describe the Householder QR factorization of A (not the process, but the properties of the output and the cost in flops).

(c) Describe any relationships between (a) and (b).

(27) 3. Let $A \in \mathbb{R}^{m \times n}, \quad m > n$ and let $b \in \mathbb{R}^m$. Let the columns of A be linearly independent. Consider the least squares problem

$$\min_x \|Ax - b\|_2 \qquad \text{(LS)}.$$

(a) Describe the normal equations approach to solving (LS). Include the cost and backward error results.

(b) Describe the Gram-Schmidt QR approach to solving (LS).

(c) Describe the Householder QR approach to solving (LS).

(d) Give conditions under which one of the above methods is clearly preferable to the others.

(9) 4. Let $x \in \mathbb{R}^n$.

(a) Compute $\|x\|_1$.

(b) Compute $\|x\|_2$.

(c) Compute $\|x\|_\infty$.

(9) 5. Let $A \in \mathbb{R}^{m \times n}$.

(a) Compute $\|A\|_1$.

(b) Compute $\|A\|_F$.

(c) How are the singular values of A related to $\|A\|_2$?