Name: ______________________________

(24) 1. Define $H \equiv H(u) = I - \frac{2}{u^t u} u u^t$.

(a) Show that $H(u) = H(\sigma u)$, for all nonzero scalars σ.

(b) If $x \in \mathbb{R}^m$ is nonzero, which vector u should we use so that $Hx = \beta e_1$?

(c) Given $u \in \mathbb{R}^m$ and $B \in \mathbb{R}^{m \times n}$, how many flops are required to compute HB?

(d) Let $u = (3, 4, 0)^t$ and let $v = (2, 3, 1)^t$. Compute Hv.

(25) 2. Let $A \in \mathbb{R}^{m \times n}, \quad m > n$ be full rank.

(a) Describe the thin QR factorization of A (not the process, but the resulting output and the properties of Q and R).

(b) Describe the explicit full QR factorization of A (not the process, but the resulting output and the properties of Q and R).

(c) The Householder QR implicit-Q factorization gives a factored Q. What does this mean?

(30) 3. Let $A \in \mathbb{R}^{m \times n}, \quad m > n$ and let $b \in \mathbb{R}^m$. Let the columns of A be linearly independent. Consider the least squares problem

$$\min_x \|Ax - b\|_2 \qquad \text{(LS)}.$$

(a) Describe the normal equations approach to solving (LS).

(b) Describe the Gram-Schmidt QR approach to solving (LS).

(c) Describe the Householder (implicit-Q) QR approach to solving (LS).

(d) What is the cost (in flops) of each of these methods?

(e) Describe the conditioning of (LS).

(21) 4. Let $A = \begin{bmatrix} -3 & 0 \\ 3 & 1 \\ 2 & 1 \end{bmatrix}$, and let $b = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

(a) Form the normal equations for these data (you don't have to solve).

(b) Find u_1 for the Householder QR factorization of A.

(c) Find q_1, the first column of the MGS QR factorization of A.

(d) Please describe the difference(s) between the classical Gram-Schmidt and modified Gram-Schmidt algorithms.