Name: ____________________________

(4) 1. State the fundamental axiom of floating point arithmetic.

(4) 2. Describe *digit cancellation* in floating point arithmetic.

(4) 3. Describe μ, the *machine epsilon*, in terms of the distances between neighboring floats.

(9) 4. Let $a = 0.0933564$ and $b = 23.23106$. Using 3 decimal digit rounding arithmetic, compute the following:

(a) $\bar{a} = \text{fl}(a)$

(b) $\bar{b} = \text{fl}(b)$

(c) What is the relative error in $\bar{a}$ as an approximation to a.

(5) 5. Count the number of flops required to multiply a $m \times m$ upper triangular matrix and an $m \times 3$ matrix.

(18) 6. Define $H \equiv H(u) = I - \frac{2}{u^t u} u u^t$, $u \neq 0$.

(a) Show that $H(u) = H(\sigma u)$, for all nonzero scalars σ.

(b) If $Hx = x$, what is the relationship between x and u?

(c) Given $u, y \in \mathbb{R}^m$, and $\beta = 2/(u^t u)$, how many flops are required to compute Hy?

(12) 7. Let $A \in \mathbb{R}^{m \times n}$, $m > n$ be full rank.

(a) Describe the explicit full QR factorization of A (not the algorithm, but the sizes and properties of Q and R, pretending you have formed Q).

(b) The Householder QR factorization gives a "factored Q". What does this mean?

(24) 8. Let $A \in \mathbb{R}^{m \times n}, \quad m > n$ and let $b \in \mathbb{R}^m$. Let the columns of A be linearly independent. Consider the least squares problem

$$\min_x \|Ax - b\|_2 \qquad \text{(LS)}.$$

(a) Describe the normal equations approach to solving (LS).

(b) Describe the Gram-Schmidt QR approach to solving (LS).

(c) Describe the Householder (factored Q) QR approach to solving (LS).

(d) What is the cost (in flops) of each of these methods?

(15) 9. Let $A = \begin{bmatrix} -5 & 1 \\ 0 & 1 \\ 12 & 0 \end{bmatrix}$, and let $b = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

(a) Form the normal equations for these data (you don't have to solve).

(b) Find q_1, the first column of the MGS QR factorization of A.

(c) Find u_1, where $H(u_1)$ is the first reflector in the HQR QR factorization of A.

(5) 10. If you had to choose between CGS and MGS to compute an orthonormal basis for a full rank matrix $A \in \mathbb{C}^{m \times n}$, which would you choose and why?