Name: ______________________________

(15) 1. Given $A = LU$ with $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) Solve $Ax = b$, where $b = (-1, 3, 9)^t$.

(b) What multiplier was used to introduce a zero at the $(3, 1)$ position of A?

(18) 2. On Norms

(a) Define the norm on $\mathbb{R}^{m \times n}$ induced by $\| \cdot \|_1$ on $\mathbb{R}^n$ and $\| \cdot \|_\infty$ on $\mathbb{R}^m$.

(b) Is $f(A) = \max_{1 \le i,j \le n} |a_{ij}|$ a norm on $\mathbb{R}^{n \times n}$? Why or why not?

(c) Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -2 & 2 \end{bmatrix}$. Compute $\|A\|_1$.

(18) 3. On Conditioning and Stability

(a) Define what it means for a method to be backward stable.

(b) Define what it means for a problem to be well conditioned.

(c) Using the ideas of conditioning and stability, describe the conditions under which we expect a computed solution to be a good approximation to the true solution.

(16) 4. Gaussian Elimination with partial pivoting (GEPP).

(a) Describe why piviting is needed in the floating point implementation of Gaussian Elimination.

(b) Describe the matrix factorization that is given by GE with partial pivoting. Carefully describe that matrices that result.

(c) What are the steps needed to solve $Ax = b$ if one already has GEPP completed?

(15) 5. Compute the $A = LU$ factorization of $A = \begin{bmatrix} -1 & 1 & 2 \\ 2 & -4 & -3 \\ -3 & 3 & 9 \end{bmatrix}$.

(18) 6. Let $\hat{x}$ be an approximate solution to $Ax = b$. Let $r = b - A\hat{x}$ be its residual, and let $e = x - \hat{x}$ be the error. Suppose the matrix and vector norms satisfy $\|Ax\| \leq \|A\|\|x\|$.

(a) Show that $\|e\| \leq \|A^{-1}\|\|r\|$.

(b) Now show that $\frac{\|e\|}{\|x\|} \leq \kappa(A)\frac{\|r\|}{\|b\|}$.

(c) Does a small residual mean $\hat{x}$ is close to x? Explain.