Name: ____________________

(28) 1. Define $H \equiv H(u) = I - \frac{2}{u^t u} u u^t$.

(a) Show that $H^2 = I$.

(b) If $x \in \mathbb{R}^m$ is nonzero, which vector u should we use so that $Hx = \alpha e_1$?

(c) Given $u \in \mathbb{R}^m$ and $B \in \mathbb{R}^{m \times n}$, how many flops are required to compute HB?

(d) Let $u = (3, 0, 1)^t$ and let $x = (2, 3, 0)^t$. Compute Hx.

(27) 2. Let $A \in \mathbb{R}^{m \times n}, \quad m > n$ be full rank.

(a) Describe the Gram-Schmidt (thin) QR factorization of A (not the process, but the resulting output and the cost in flops).

(b) Describe the Householder (full) QR factorization of A (not the process, but the resulting output and the cost in flops).

(c) Compare and contrast the two factorizations.

(27) 3. Let $A \in \mathbb{R}^{m \times n}$, $m > n$ and let $b \in \mathbb{R}^m$. Let the columns of A be linearly independent. Consider the least squares problem

$$\min_x \|Ax - b\|_2 \qquad \text{(LS)}.$$

(a) Describe the normal equations approach to solving (LS).

(b) Describe the Gram-Schmidt QR approach to solving (LS).

(c) Describe the Householder QR approach to solving (LS).

(d) Which method is fastest, and what is that flop count?

(18) 4. Let $A \in \mathbb{R}^{n \times n}$ be nonsingular and let $b \in \mathbb{R}^n$. For any $x \in \mathbb{R}^n$, define the residual $r = b - Ax$. (Don't make this hard: A is nonsingular and the question is about how (LS) meets $Ax = b$).

(a) What is the minimum possible value for $\|r\|_2$ in this case?

(b) What value of x gives this minimum value?

(c) Should the normal equations be used to compute x here? Why or why not?

(d) What method would you use in this case? Why? (There are lots of correct answers here, you will be graded mostly on the 'Why?' part).