Name: _____

(30) 1. Let $A \in \mathbb{R}^{m \times n}$ be full rank with $m \geq n$.

(a) Discuss the similarities and differences between the Gram-Schmidt QR factorization and the Householder QR factorization. Restrict your discussion to the mathematics of the factorizations, not the actual implementations.

(b) Now discuss the output of our QR implementations. That is, What output does each method above give (and how is that output related to the factorizations described above) and what is the cost of each?

(40) 2. Let $A \in \mathbb{R}^{m \times n}$, $m > n$ and let $b \in \mathbb{R}^m$. Let the columns of A be linearly independent. Consider the least squares problem

$$\min_x \|Ax - b\|_2. \qquad (LS)$$

How do we find x (very coarse pseudocode) using

(a) The normal equations approach?

(b) MGS QR approach?

(c) Householder QR approach?

(d) What is cost (flop count) of each of the above approaches?

(30) 3. Miscellaneous

(a) Define a Householder reflector, and show that it is orthogonal.

(b) Let $u \in \mathbb{R}^m$, and $H = H(u)$ be a Householder reflector. How many flops are required to compute $y = Hx$?

(c) Describe the primary difference between the computed Q from modified Gram-Schmidt and the computed Q from classical Gram-Schmidt.

(d) Show that if x_{LS} is the solution to $min_x \|b - Ax\|_2$, then the residual $r = b - Ax_{LS}$ is orthogonal to ColSp(A).