Name: _____

(28) 1. Define $H \equiv H(u) = I - \frac{2}{u^t u} uu^t$.

(a) Show that $H^2 = I$.

(b) If $x \in \mathbb{R}^m$ is nonzero, which vector u should we use so that $Hx = \alpha e_1$?

(c) Given $u \in \mathbb{R}^m$ and $B \in \mathbb{R}^{m \times n}$, how many flops is required to compute HB?

(d) Let $u = (0, 3, 1)^t$ and let $x = (2, 3, 0)^t$. Compute Hx.

(27) 2. Let $A \in \mathbb{R}^{m \times n}$, $m > n$ be full rank.

(a) Describe the Gram-Schmidt QR factorization of A (not the process, but the resulting output and the cost in flops).

(b) Describe the Householder QR factorization of A (not the process, but the resulting output and the cost in flops).

(c) Describe any relationships between (a) and (b).

(27) 3. Let $A \in \mathbb{R}^{m \times n}, \quad m > n$ and let $b \in \mathbb{R}^m$. Let the columns of A be linearly independent. Consider the least squares problem

$$\min_x \|Ax - b\|_2 \qquad \text{(LS)}.$$

(a) Describe the normal equations approach to solving (LS).

(b) Describe the Gram-Schmidt QR approach to solving (LS).

(c) Describe the Householder QR approach to solving (LS).

(d) Give conditions under which one of the above methods is clearly preferable to the others.

(18) 4. Let $A \in \mathbb{R}^{n \times n}$ be nonsingular and let $b \in \mathbb{R}^n$. For any $x \in \mathbb{R}^n$, define the residual $r = b - Ax$.

(a) What is the minimum possible value for $\|r\|_2$ in this case?

(b) What value of x gives this minimum value?

(c) What are the normal equations in this case?

(d) Should the normal equations be used in this case? Why or why not?

(e) What is the orthogonal projector for $\text{ColSp}(A)$ in this case?