Name: ____________________

(18) 1. On Conditioning and Stability

(a) Define *backward stable.*

(b) Define *well conditioned.*

(c) Using the ideas of conditioning and backward stability, describe the conditions for which a computed solution to be a good approximation to the true solution.

(18) 2. Define $H \equiv H(u) = I - \frac{2}{u^t u} u u^t$.

(a) Show that $H(u) = H(\sigma u)$, for all nonzero scalars σ.

(b) If $x, y \in \mathbb{R}^m$ are nonzero, $\beta \in \mathbb{R}$, and $Hx = \beta y$, then what is $|\beta|$?

(c) Given $u, x \in \mathbb{R}^m$, how many flops are required to compute Hx?

(19) 3. Let $A \in \mathbb{R}^{m \times n}, \quad m > n$ be full rank.

(a) Describe the thin QR factorization of A (not the process, but the resulting output and the properties of Q and R).

(b) Describe the explicit full QR factorization of A (not the process, but the resulting output and the properties of Q and R).

(c) The Householder QR factorization gives a "factored Q". What does this mean?

(24) 4. Let $A \in \mathbb{R}^{m \times n}, \quad m > n$ and let $b \in \mathbb{R}^m$. Let the columns of A be linearly independent. Consider the least squares problem

$$\min_x \|Ax - b\|_2 \qquad \text{(LS)}.$$

(a) Describe the normal equations approach to solving (LS).

(b) Describe the Gram-Schmidt QR approach to solving (LS).

(c) Describe the Householder (factored Q) QR approach to solving (LS).

(d) What is the cost (in flops) of each of these methods?

(15) 5. Let $A = \begin{bmatrix} -4 & 0 \\ 3 & 1 \\ 0 & 1 \end{bmatrix}$, and let $b = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

(a) Form the normal equations for these data (you don't have to solve).

(b) Find u_1 for the Householder QR factorization of A.

(c) Find q_1, the first column of the MGS QR factorization of A.

(6) 6. Say what you can about either the SVD or "Conditioning of (LS)".