Math 4353 – Numerical Linear Algebra   Spring nnnn   Test 1

Name: ______________________________

(24) 1. On Floating Point Arithmetic

(a) Carefully state the fundamental axiom of floating point arithmetic (FAFA).

(b) Describe digit cancellation.

(c) Does digit cancellation violate the FAFA? Why or why not?

(d) In the following use 3 decimal digit rounding arithmetic.

i. If $x = 123.456$. What is $\mathrm{fl}(x)$?

ii. If $y = 0.00987654$. What is $\mathrm{fl}(y)$?

iii. What is the "machine precision" for this arithmetic?

(18) 2. On Norms

(a) Define a vector norm on the vector space $\mathbb{R}^n$.

(b) Given a vector norm $\|\cdot\|_v$, define the matrix norm on $\mathbb{R}^{n\times n}$ induced by $\|\cdot\|_v$.

(c) Let $x = [2, 0, -3]$. Compute $\|x\|_2$ and $\|x\|_1$.

(12) 3. Give an algorithm for multiplying an $n \times n$ lower triangular matrix and an n-vector. Give the number of flops that this algorithm requires.

(12) 4. Let $A = \begin{bmatrix} -9 & -8 & -7 & -6 & -5 \\ -4 & -3 & -2 & -1 & 0 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$. Let $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$,

be a partitioning (blocking) of A, where A_{11} is 2×2 and A_{23} is 1×2.

(a) What is A_{12}?

(b) If $B = [1, 3, 5, 7, 9]^t$, define a partitioning of B conformal with multiplication on the left by A.

(c) If $C = AB$, then what is the partition of C defined by the above partitions?

(9) 5. Matrix multiplication is associative, but some algorithms are more efficient than others. Consider the computation of the product $ABCD$, when $A \in \mathbb{R}^{n \times 1}$, $B \in \mathbb{R}^{1 \times n}$, $C \in \mathbb{R}^{n \times 1}$, and $D \in \mathbb{R}^{1 \times n}$. Estimate the number of flops for each of the following algorithms:

(a) $((AB)C)D$

(b) $(AB)(CD)$

(c) $A((BC)D)$

(25) 6. Let $A = U\Sigma V^t \in \mathbb{R}^{5\times 4}$ be a singular value decomposition where $U = [u_1, u_2, \ldots, u_5]$, $V = [v_1, v_2, \ldots, v_4]$, and $\Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_p)$. Let $\sigma_1 = 1$, $\sigma_2 = 1/4$ and $\sigma_3 = 0$.

(a) What is the rank of A?

(b) What is the dimension of the nullspace of A?

(c) Give an orthonormal basis for the range of A:

(d) Give an orthonormal basis for the nullspace of A:

(e) Write numbers, not variables: $\Sigma =?$

(f) Write numbers, not variables: $VV^t = ?$