Math 4353 – Numerical Linear Algebra Spring nnnn Test 1

Name: ______________________________

(24) 1. On Floating Point Arithmetic

(a) Carefully define underflow and overflow.

(b) Carefully state the fundamental axiom of floating point arithmetic (FAFA).

(c) Describe swamping (create a simple example if you like).

(d) In the following use 4 decimal digit rounding arithmetic.

i. If $x = 124.364$. What is $\mathrm{fl}(x)$?

ii. If $y = 0.00876441$. What is $\mathrm{fl}(y)$?

(18) 2. Conditioning and Stability

(a) Define a *backward stable* computation.

(b) Define an *illconditioned* problem.

(c) Now use the ideas above to describe under what conditions we can expect a computed solution to have small relative error.

(11) 3. Explain (i) *why* we use pivoting in Gaussian Elimination, and (ii) discuss the backward stability of GEPP.

(3) 4. Let $A = \begin{bmatrix} -9 & -8 & -7 \\ -6 & -5 & -4 \\ -3 & -2 & -1 \\ 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{33} \end{bmatrix}$ be a partitioning (blocking) of A, where A_{11} is 2×2 and A_{33} is 1×1.

(a) What is A_{22}?

(18) 5. Let $A \in \mathbb{R}^{n \times n}$, $u, v \in \mathbb{R}^{n \times 1}$, and I be the identity matrix.

(a) Count the number of flops for the following algorithm:
1. Compute $W = uv^t$
2. Compute $B = I + W$
3. Compute $Z = BA$

(b) Give Z in terms of A, I, u, and v.

(c) Give a faster algorithm for computing Z and give its flop count. (Don't go into details, but mimic part (a) above.)

(9) 6. Let $A \in \mathbb{R}^{3\times n} = [a_1, a_2, a_3]^t$, let $m_1 = (0, 2, -1)^t$ and let e_k be the k^{th} column of the identity matrix. Let $B = (I + m_1 e_1^t)A$.

(a) What is the first row of B?

(b) What is the second row of B?

(c) What is the third row of B? (only worth 1 point)

(7) 7. Let $x = (2, -1, 4)^t$.

(a) What should m be if $(I + me_1^t)x = (2, 0, 0)^t$?

(b) What should m be if $(I + me_2^t)x = (0, -1, 0)^t$? (only worth 1 point)

(10) 8. Suppose we are given L and U in the LU-decomposition of A. Describe L and U and show how can we use them to solve $Ax = b$.