Name: _____

(5) 1. Define *swamping* in floating point arithmetic.

(5) 2. Define *digit cancellation* in floating point arithmetic.

(9) 3. Let $a = 0.0623446$ and $b = 13.22601$. Using 4 decimal digit rounding arithmetic, compute the following:

(a) $\text{fl}(a)$

(b) $\text{fl}(b)$

(c) What is the relative error in $\bar{a}$

(3) 4. What is *underflow*?

(4) 5. State the *floating point representation theorem*. (That one says how much error $\text{fl}(x)$ can have).

(25) 6. On Conditioning and Stability

(a) What is a well conditioned problem?

(b) Describe what a condition number is.

(c) What is a backward stable computation?

(d) How can we use the ideas of conditioning and stability to assess the error in a computation?

(27) 7. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 14 & 10 & 1 \\ 4 & 2 & -1 \end{bmatrix}$.

(a) Give L and U from the $A = LU$ factorization of A.

(b) Explain how pivoting effects the multipliers in Gaussian elimination.

(c) Why is pivoting used in Gaussian elimination?

(10) 8. Solve $Ax = b$, where $PA = LU$ and

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad \text{and } b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

(6) 9. What is the relationship, if any, between the machine epsilon and the distance between floating point numbers.

(6) 10. Count the number of flops required to multiply a $n \times n$ lower triangular matrix and an n-vector.