Name: ____________________________

(5) 1. Define *backward stability* for a computation.

(5) 2. Define *digit cancellation* in floating point arithmetic.

(4) 3. Describe the *machine epsilon* in terms of the distance between neighboring floats.

(5) 4. Do you agree or disagree with the statement "if a computation is backward stable, then the computed solution is close to the true solution"? Explain.

(9) 5. Let $a = 0.0933464$ and $b = 23.26106$. Using 4 decimal digit rounding arithmetic, compute the following:

(a) $\bar{a} = \mathrm{fl}(a)$

(b) $\bar{b} = \mathrm{fl}(b)$

(c) What is the relative error in $\bar{a}$ as an approximation to a.

(15) 6. Gauss transforms: let $A \in \mathbb{R}^{n \times n}$ and $m_k \in \mathbb{R}^n$ have 0's in its first k entries (and e_j is the jth column of I).

(a) How many flops are required to compute $B = (I + m_1 e_1^t)A$?

(b) How many flops are required to compute $B = (I + m_k e_k^t)A$?

(c) What is the inverse of $I + m_k e_k^t$? Justify your answer.

(15) 7. Describe how we can use the factorization $PA = LU$ to solve the square system $Ax = b$. Give the flop count for your method.

(27) 8. Let $A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & 0 & 0 \\ -2 & -1 & 3 \end{bmatrix}$.

(a) Give L and U from the $A = LU$ factorization of A.

(b) Pivoting in Gaussian elimination (GEPP) guarantees what fact about the multipliers?

(c) *Why* is pivoting used in Gaussian elimination?

(8) 9. Solve $Ax = b$, where $PA = LU$ and

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \quad \text{and } b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

(7) 10. Count the number of flops required to multiply a $n \times n$ upper triangular matrix and an $n \times 2$ matrix.