Name: ____________________

(5) 1. Assume that x, y and $x + y$ are real numbers in the floating point range. Show that $\mathrm{fl}(x + y)$ is backward stable.

(5) 2. Define *digit cancellation* in floating point arithmetic.

(9) 3. Let $a = 0.00123701$ and $b = 1234.01$. Using 3 decimal digit rounding arithmetic, compute the following:

(a) $\bar{a} = \mathrm{fl}(a)$

(b) $\bar{b} = \mathrm{fl}(b)$

(c) $\bar{c} = \mathrm{fl}(\bar{b} + \bar{a})$

(4) 4. For a floating point system with machine epsilon μ, what is the maximum relative difference between 2 neighboring positive floats?

(4) 5. State the fundamental axiom of floating point arithmetic.

(20) 6. On Conditioning and Stability

(a) What is a well conditioned problem?

(b) What does a condition number measure?

(c) What is a backward stable computation?

(d) How can we use the ideas of conditioning and stability to evaluate the error in a computation?

(12) 7. Let $A = \begin{bmatrix} 2 & 0 & 2 \\ 4 & 3 & 5 \\ 0 & -9 & -2 \end{bmatrix}$.

8. Give L and U from the $A = LU$ factorization of A.

(12) 9. Let $A \in \mathbb{R}^{n \times n}$ and $A = LU$ and $PA = L'U'$ be the factorizations given by G.E. with no pivoting, and partial pivoting, resp.

(a) Give a formula for $e_i^t L e_1$.

(b) Give a bound for $e_i^t L' e_1$.

(c) Explain how a |small| diagonal element, $a_{kk}^{(k-1)}$, adversely effects the Gaussian elimination process if no pivoting is used.

(13) 10. Solve $Ax = b$, where $PA = LU$ and

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}, \quad \text{and } b = \begin{bmatrix} 4 \\ 6 \end{bmatrix}.$$

(8) 11. Let $\bar{x}$ be a computed solution to $Ax = b$ and $r = b - A\bar{x}$ be the residual. Show that if $\|Ax\| \leq \|A\|\|x\|$, then

$$\frac{\|x - \bar{x}\|}{\|x\|} \leq \kappa(A)\frac{\|r\|}{\|b\|}.$$

(8) 12. If A is $n \times n$ and u and v are $n \times 1$, then how many flops are required to compute:

(a) $(uv^t)A$?

(b) $u(v^tA)$?