Name: ______________________________

(5) 1. Define *swamping* in floating point arithmetic.

(5) 2. Define *digit cancellation* in floating point arithmetic.

(12) 3. Let $a = 0.0123601$ and $b = 1234.01$. Using 3 decimal digit rounding arithmetic, compute the following:

(a) $\bar{a} = \mathrm{fl}(a)$

(b) $\bar{b} = \mathrm{fl}(b)$

(c) The relative error in $\bar{b}$ (you can round to 2 significant digits).

(5) 4. How is the unit round-off, μ, related to the distance between neighboring floats?

(4) 5. State the fundamental axiom of floating point arithmetic (don't forget the hypotheses).

(5) 6. Describe what we mean by a *backward stable computation.*

(12) 7. Assume A is nonsingular. Let x be the true solution to $Ax = b$ and let $\bar{x}$ be a computed approximation to x.

(a) What is the residual vector, $\bar{r}$, associated with $\bar{x}$?

(b) What can $\bar{r}$ tell us about backward stability of $\bar{x}$?

(c) What is the error vector, $x - \bar{x}$, in terms of $\bar{r}$ and A^{-1}?

(25) 8. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -5 & -2 \\ 2 & 5 & 4 \end{bmatrix}$.

(a) Give L and U from the $A = LU$ (no pivoting) factorization of A.

(b) Explain how pivoting effects the multipliers.

(c) Explain how a |small| pivot element, $a_{kk}^{(k-1)}$, adversely effects the Gaussian elimination process.

(10) 9. Solve $Ax = b$, where $PA = LU$ and

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \quad \text{and } b = \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

(10) 10. Let $A \in \mathbb{R}^{n \times n}$. How many flops are required to...

(a) compute the LU factorization (Gaussian elimination) of A?

(b) solve $Ly = b$?

(7) 11. If A is $n \times n$ and u and v are $n \times 1$, then how many flops are required to compute:

(a) $(uv^t)A$?

(b) $u(v^t A)$?