Name: _____

(16) 1. Suppose $A \in \mathbb{C}^{m\times n}$, $B \in \mathbb{C}^{n\times p}$, and $C = AB$.

(a) If $b_j = Be_j$ is the j^{th} column of B, then what is Ce_j?

(b) If $\alpha_i^* = e_i^* A$ is the i^{th} row of A, then what is $e_i^* C$?

(c) Using the α's and b's from above, what is $c_{ij} = e_i^* C e_j$?

(d) If $a_j = Ae_j$ is the j^{th} column of A, and $\beta_i^* = e_i^* B$ is the i^{th} row of B, then give C in terms of the a's and β's.

(16) 2. Let $< x, y >= x^* y$ be the standard inner product on $\mathbb{C}^m$ and let $Q \in \mathbb{C}^{m\times m}$ be unitary.

(a) What is the solution, x, to the system of equations $Qx = b$?

(b) If $Q = [q_1, q_2, \ldots, q_m]$, and $b = \sum_{j=1}^{n} c_j q_j$, then what is c_j?

(c) What is the solution, X, to the matrix equation $XQ = B$?

(d) What is the solution, X, to the matrix equation $XQ = B$?

(5) 3. Let $\|\cdot\|_R$ is a vector norm on $\mathbb{C}^m$ and $\|\cdot\|_D$ is a vector norm on $\mathbb{C}^n$, write the definition of the norm for a matrix $A \in \mathbb{C}^{m\times n}$ induced by these vector norms.

(9) 4. Let $x = [-1,\ 1, -2]^*$.

(a) What is $\|x\|_1$?

(b) What is $\|x\|_2$?

(c) What is $\|x\|_\infty$?

(12) 5. Let $A = \begin{bmatrix} 3 & -4 \\ 2 & 0 \\ -5 & 5 \end{bmatrix}$.

(a) What is $\|A\|_1$?

(b) What is $\|A\|_F$?

(c) What is $\|A\|_\infty$?

(6) 6. Describe the singular value decomposition of a matrix $A \in \mathbb{C}^{m \times n}$. (give the factorization, and properties of the factors).

(12) 7. If A is a 4 x 4 matrix with singular values $\sigma_1 = 2$, $\sigma_2 = 2$, $\sigma_3 = 2$, and $\sigma_4 = 0$, then

(a) What is $\|A\|_2$?

(b) What is $\|A\|_F$?

(c) is A singular or nonsingular? How do you know?

(12) 8. Let S be a subspace of $\mathbb{C}^m$, and the vectors $q_1, q_2, \ldots, q_n$ be an orthonormal basis for S.

(a) Give a formula for the orthonormal projector, P, onto S.

(b) If $b \in \mathbb{C}^m$, what vector in S is the closest (in 2-norm) to b?

(c) If y is the vector above (nearest in S to b), what is $(b - y)^* y$)?

(d) If $y \in S$, what is Py?

(12) 9. Let $A = QR \in \mathbb{C}^{m \times n}$ is a reduced (Gram-Schmidt) QR factorization.

(a) Describe the sizes and properties of Q and R.

(b) Outline a method that uses this $A = QR$ factorization to compute the solution, x, to the least squares problem

$$\operatorname*{argmin}_x \|b - Ax\|_2.$$

(You do not need to describe the details of your Gram-Schmidt algorithm)