Math 4353 – Numerical Linear Algebra Spring nnnn Test 1

Name: _____

(24) 1. On Floating Point Arithmetic

(a) Carefully define underflow and overflow.

(b) Carefully state the fundamental axiom of floating point arithmetic (FAFA).

(c) Describe digit cancellation.

(d) In the following use 4 decimal digit rounding arithmetic.

i. If $x = 124.346$. What is $\text{fl}(x)$?

ii. If $y = 0.00876451$. What is $\text{fl}(y)$?

(18) 2. On Norms

(a) Define a vector norm on the vector space $\mathbb{R}^n$.

(b) Is $f(x) = x^t x$ a norm on $\mathbb{R}^n$? Why or why not?

(c) Let $x = [1, 2, -3]$. Compute $\|x\|_1$, $\|x\|_2$, and $\|x\|_\infty$.

(8) 3. Let $L, M \in \mathbb{R}^{n \times n}$ be lower triangular. Show that LM is lower triangular.

(6) 4. Let $A = \begin{bmatrix} -9 & -8 & -7 \\ -6 & -5 & -4 \\ -3 & -2 & -1 \\ 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{33} \end{bmatrix}$ be a partitioning (blocking) of A, where A_{11} is 2×1 and A_{33} is 2×2.

(a) What is A_{22}?

(b) What is A_{21}^t?

(18) 5. Let $A \in \mathbb{R}^{n \times n}$, $u, v \in \mathbb{R}^{n \times 1}$, and I be the identity matrix.

(a) Count the number of flops for the following algorithm:
1. Compute $W = uv^t$
2. Compute $B = I + W$
3. Compute $Z = BA$

(b) Give Z in terms of A, I, u, and v.

(c) Give a faster algorithm for computing Z and give its flop count. (Don't go into details, but do like in part (a) above.)

(9) 6. Let $A \in \mathbb{R}^{3\times n} = [a_1, a_2, a_3]^t$, let $m_1 = (0, 2, -1)^t$ and let e_k be the k^{th} column of the identity matrix. Let $B = (I + m_1 e_1^t)A$.

(a) What is the first row of B?

(b) What is the second row of B?

(c) What is the third row of B? (only worth 1 point)

(7) 7. Let $x = (2, -1, 4)^t$.

(a) What should m be if $(I + me_1^t)x = (2, 0, 0)^t$?

(b) What should m be if $(I + me_2^t)x = (0, -1, 0)^t$? (only worth 1 point)

(10) 8. Suppose we are given L and U in the LU-decomposition of A. Describe L and U and show how can we use them to solve $Ax = b$.