Numerical Linear Algebra Spring nnnn Test 1

Name: _____

(5) 1. What is *digit cancellation* in floating point arithmetic.

(5) 2. What is *underflow* in floating point arithmetic?

(5) 3. State the *fundamental axiom of floating point arithmetic*. (That one is about the error in $\mathrm{fl}(x \,\square\, y)$. Don't forget to include the hypotheses).

(5) 4. What is the relationship, if any, between the machine epsilon and the distance between floating point numbers.

(15) 5. Let $a = 0.0923436$ and $b = 732.2791$. Using 4 decimal digit rounding arithmetic, compute the following:

(a) $\bar{a} = \text{fl}(a)$

(b) $\bar{b} = \text{fl}(b)$

(c) What is the absolute error in $\bar{b}$

(7) 6. Derive an upper bound on the relative error in computing $\text{fl}(ab)$, where $a, b, ab \in \mathbb{R}$ do not underflow or overflow, and $\boldsymbol{\mu}$ is the machine epsilon.

(8) 7. Suppose $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times p}$, and $C = AB$. Let e_k be the k^{th} column of I.

(a) What is c_{ij} in terms of A, B and the e_k's?

(b) Using A, B and the e_k's, write C as a sum of rank 1 (outer-product) matrices.

(26) 8. Let $A = \begin{bmatrix} 2 & 3 & 5 \\ -2 & -2 & -6 \\ 4 & 6 & 8 \end{bmatrix}$.

(a) Give L and U from the $A = LU$ factorization of A.

(b) Pivoting in Gaussian elimination (GEPP) guarantees what fact about the multipliers?

(c) Why do we want to avoid large multipliers?

(8) 9. If $A \in \mathbb{R}^{n\times n}$ and $u, v \in \mathbb{R}^{n\times 1}$, then how many flops are required to compute:

(a) $(uv^t)A$?

(b) $u(v^tA)$?

(6) 10. Let $A \in \mathbb{R}^{n\times n}$ have rows $e_i^tA = a_i^t$, let $m \in \mathbb{R}^n$ and let e_k be the k^{th} column of the identity matrix. Let $B = (I + me_k^t)A$.

What is the j^{th} row of B (in terms of the elements of m and the rows of A)?

(10) 11. Suppose we are given L and U in the LU factorization of a nonsingular $A \in \mathbb{R}^{n\times n}$.

(a) Describe L and U

(b) Show how we use L and U to solve $Ax = b$.