Math 4353 – Numerical Linear Algebra Spring nnnn Test 1

Name: _____

(14) 1. Let $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times p}$, and $C \in \mathbb{R}^{p \times q}$.

(a) How many flops are needed to compute AB?

(b) Now adjust your answer from part (a) to answer the following:

i. How many flops are needed to compute BC?

ii. How many flops are needed to compute $(AB)C$?

iii. How many flops are needed to compute $A(BC)$?

(21) 2. Norms, inner products and orthogonality.

(a) Give the definition of a norm on a vector space.

(b) Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -2 \end{bmatrix}$. Compute $\|A\|_1$, $\|A\|_F$, and $\|A\|_\infty$.

(c) How is an inner product (dot product) related to norms and angles in $\mathbb{C}^n$? (Give a formula).

(d) What does it mean for 2 subspaces, say S and T, to be orthogonal?

(20) 3. Define the singular value decomposition for a square matrix $A \in \mathbb{C}^{n \times n}$. Make sure you give the factorization and the necessary properties of the matrices and the singular values.

(12) 4. Gram-Schmidt

(a) Here is a picture of two vectors, a_1 and a_2 in $\mathbb{R}^2$. Draw q_1 and q_2, and label the quantities r_{11}, r_{12} and r_{22} in the Gram-Schmidt orthogonalization of a_1 and a_2.

(b) Using the quantities above, write a_2 as a linear combination of q_1 and q_2.

(15) 5. Householder Matrices

(a) Give the definition of a Householder matrix.

(b) Show that Householder matrices are unitary.

(c) The Householder QR factorization does not explicitly give the matrix Q. What does it give, and how is that related to Q?

(18) 6. Least squares: $\min_x \|Ax - b\|_2$ (LS).

(a) If $S = \text{Range}(A)$, describe the solution of (LS) in terms of the orthogonal projector P for S.

(b) Write down the normal equations for (LS).

(c) Explain how to use a QR factorization to find x.