Name: _____

(4) 1. Describe *swamping* in floating point arithmetic.

(4) 2. Describe *digit cancellation* (cancellation) in floating point arithmetic.

(12) 3. Let $a = 0.0123401$ and $b = 1234.51$. Using 3 decimal digit rounding arithmetic, answer the following:

(a) What is the value of $\bar{a} = \text{fl}(a)$?

(b) What is the value of $\bar{b} = \text{fl}(b)$?

(c) Give the relative error in $\bar{b}$ (you can round to 2 significant digits).

(d) What would this (3 decimal digit) computer return when evaluating c = (a + b) - b?

(4) 4. How is the unit round-off, $\boldsymbol{\mu}$, related to the distance between neighboring floats?

(4) 5. Describe the differences, if any, between $\boldsymbol{\mu}$ and the smallest positive floating point number.

(4) 6. State the fundamental axiom of floating point arithmetic (don't forget the hypotheses).

(4) 7. Describe what we mean by a *backward stable computation*.

(9) 8. Let $x = (2, -4, 3)^T$. Compute $\|x\|_1$, $\|x\|_2$, and $\|x\|_\infty$.

(9) 9. Let $A = \begin{bmatrix} 0 & 1 & -3 \\ -5 & 0 & 1 \end{bmatrix}$. Compute $\|A\|_1$, $\|A\|_\infty$, and $\|A\|_F$.

(22) 10. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & -9 & -2 \\ 2 & 5 & 4 \end{bmatrix}$.

(a) Give L and U from the $A = LU$ (no pivoting) factorization of A.

(b) Explain how a |small| pivot element, $a_{kk}^{(k-1)}$, adversely effects the Gaussian elimination process.

(c) Explain how row pivoting effects the multipliers (include any bounds on $|m_{i,j}|$ associated with pivoting).

(8) 11. Solve $Ax = b$, given $PA = LU$ and

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \quad \text{and } b = \begin{bmatrix} 5 \\ -3 \end{bmatrix}.$$

(8) 12. Let $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$. How many flops are required to... (you need only give the leading term and you don't need to derive/prove)

(a) compute the LU factorization (Gaussian elimination) of A?

(b) solve $Ly = b$?

(8) 13. If A is $n \times n$ and u and v are $n \times 1$, then how many flops are required to compute:

(a) $(uv^T)A$?

(b) $u(v^T A)$?