Name: _____

(5) 1. Define *swamping* in floating point arithmetic.

(5) 2. Define *digit cancellation* in floating point arithmetic.

(9) 3. Let $a = 0.0123401$ and $b = 1236.01$. Using 3 decimal digit rounding arithmetic, compute the following:

(a) $\bar{a} = \text{fl}(a)$

(b) $\bar{b} = \text{fl}(b)$

(c) $\bar{c} = \text{fl}(\bar{a} + \bar{b})$

(3) 4. How is the machine precision, μ, related to the distance between neighboring floats?

(4) 5. State the fundamental axiom of floating point arithmetic.

(25) 6. On Conditioning and Stability

(a) What is a well conditioned problem?

(b) Describe what a relative condition number is.

(c) What is a backward stable computation?

(d) How can we use the ideas of conditioning and stability to evaluate the error in a computation?

(27) 7. Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 4 & -1 & 3 \\ -6 & -1 & 8 \end{bmatrix}$.

(a) Give L and U from the $A = LU$ factorization of A.

(b) Explain how pivoting effects the multipliers.

(c) Explain how a |small| pivot element, $a_{kk}^{(k-1)}$, adversely effects the Gaussian elimination process.

(10) 8. Solve $Ax = b$, where $PA = LU$ and

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \quad \text{and } b = \begin{bmatrix} 5 \\ 2 \end{bmatrix}.$$

(6) 9. Let $\bar{x}$ be a computed solution to $Ax = b$ and $r = b - A\bar{x}$ be the residual. Describe in plain English the following

$$\frac{\|A^{-1}r\|}{\|x\|}.$$

(6) 10. If A is $n \times n$ and u and v are $n \times 1$, then how many flops are required to compute:

(a) $(uv^t)A$?

(b) $u(v^tA)$?