Numerical Linear Algebra | Spring nnnn | Test 1

Name: ______________________________

Recall that we use the notation: e_k is the k^{th} column of I.

(5) 1. What is *swamping* in floating point arithmetic?

(5) 2. What is *underflow* in floating point arithmetic?

(5) 3. State the *fundamental axiom of floating point arithmetic*. (That one is about the error in $\mathrm{fl}(x \,\square\, y)$. Don't forget to include the hypotheses).

(9) 4. Let z be a positive floating point number such that $\mathrm{fl}(z+1) = z$.

(a) What can be said about z?

(b) What is $\mathrm{fl}(\mathrm{fl}(1+z) - z)$?

(c) What is $\mathrm{fl}(1 + \mathrm{fl}(z - z))$?

(12) 5. Let $a = 0.00123463$ and $b = 732.2179$. Using 4 decimal digit rounding arithmetic, compute the following:

(a) $\bar{a} = \text{fl}(a)$

(b) $\bar{b} = \text{fl}(b)$

(c) The relative error in $\bar{a}$

(12) 6. Let $A = \begin{bmatrix} -4 & 3 & 2 \\ 3 & -2 & 3 \end{bmatrix}$. Compute the following:

(a) $\|Ae_2\|_2$

(b) $\|A\|_1$

(c) $\|A\|_\infty$

(d) $\|A\|_F$

(10) 7. Suppose $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times p}$, and $C = AB$.

(a) What is c_{ij} in terms of A, B and the e_k's?

(b) Using A, B and the e_k's, write C as a sum of rank 1 (outer-product) matrices.

(21) 8. Let $A = \begin{bmatrix} 2 & 3 & 5 \\ -2 & -2 & -7 \\ 2 & 3 & 3 \end{bmatrix}$.

(a) Give L and U from the $A = LU$ factorization of A.

(b) Pivoting in Gaussian elimination (GEPP) guarantees what fact about the multipliers?

(c) Why do we want to avoid large multipliers?

(6) 9. Let $A \in \mathbb{R}^{n \times n}$ have rows $e_i^t A = a_i^t$ and let $m \in \mathbb{R}^n$. Let $B = (I + me_k^t)A$.

What is the j^{th} row of B (in terms of the elements of m and the rows of A)?

(15) 10. Suppose we are given L and U in the LU factorization of a nonsingular $A \in \mathbb{R}^{n \times n}$.

(a) Carefully Describe the structure of L and U

(b) Explain how we use L and U to solve $Ax = b$.

(c) If we had $PA = LU$ with P nonsingular, explain how we use P, L, and U to solve $Ax = b$.