Name: _____

(5) 1. Carefully state the floating point representation theorem.

(5) 2. Carefully state the fundamental axiom of floating point arithmetic.

(6) 3. Let $a = 0.00123601$ and $b = 12360.1$. Using 4 decimal digit rounding arithmetic, compute the following:

(a) $\text{fl}(a)$

(b) $\text{fl}(b)$

(10) 4. Consider the computation of $c = ab$, where a and b are real numbers (not necessarily floats). Let the *computed* product be $\bar{c}$, and assume that none of a, b or c overflow or underflow. Give a bound for the relative error in $\bar{c}$.

(25) 5. On Conditioning and Stability

(a) Define ill-conditioning

(b) Describe what a condition number tells us.

(c) What is a backward stable computation?

(d) What is a backward stable method?

(e) Discuss how conditioning and stability can be used to evaluate the error in a computation.

(27) 6. Let $A = \begin{bmatrix} 1 & 9 \\ 6 & 3 \end{bmatrix}$. Let ρ be the growth factor for Gaussian elimination.

(a) Give L, U, and ρ from the $A = LU$ factorization of A.

(b) Give L, U, P, and ρ from the $PA = LU$ factorization of A.

(c) Explain why pivoting is used in Gaussian Elimination and give an example of an invertible matrix that does not have an LU factorization.

(12) 7. Let $A \in \mathbb{R}^{n \times n}$ be nonsingular and $b \in \mathbb{R}^n$. Describe how the $PA = LU$ factorization can be used to solve the system $Ax = b$. Give a flop count for each step.

(10) 8. Find the number of multiplications required to solve an $n \times n$ lower triangular system using forward substitution.