Arnold problems for homework 1

1. Let (a) be the statement

$$\frac{|\bar{x} - x|}{|x|} \leq \mu \qquad \text{(a)},$$

   and (b) be the statement

$$\bar{x} = x(1 + \delta), \; |\delta| \leq \mu \qquad \text{(b)}.$$

   Show that for nonzero $x \in \mathbb{R}$, (a) is true if and only if (b) is true.

2. Let $x = [4.0091, 0.12319, 1.2341]^T$ and $y = [-1.1021, .35449, 3.5449]^T$. Using 3 decimal-digit arithmetic compute $\bar{x} = \text{fl}(x)$, $\bar{y} = \text{fl}(y)$, $c = \text{fl}(\bar{x}^T\bar{x})$ and $d = \text{fl}(\bar{x}^T\bar{y})$.

   Now compute the actual values of $x^T x$ and $x^T y$ (to 5 or 6 significant digits) and find the relative errors

$$\frac{|x^T x - c|}{|x^T x|}$$

   and

$$\frac{|x^T y - d|}{|x^T y|}.$$

   Note: $s = \text{fl}(u^T v)$ for $u, v \in \mathbb{R}^n$ is to be computed in the standard way (the same way as the textbook problem in this homework assignment):

   s=0; for j=1:n, $s = fl(s + fl(u_j * v_j))$; end

3. For 53903 students: Look up 'Kahan summation algorithm' and 'pairwise summation'. These are more accurate (but slower) ways to sum (or compute a dot product).