Notation and Language

If S is a set, then $x \in S$ means x is a member of S (x is 'in' S).
$\forall x \in S$ means 'for all' or 'for every' $x \in S$.
$\exists x \in S$ means 'there exists' $x \in S$.
$S \subset T$ means if $x \in S$, then $x \in T$. We say S is a subset of T .

$P \Rightarrow Q$ means 'if statement P is true, then statement Q is true'.
$Q \Rightarrow P$ is the *converse* of $P \Rightarrow Q$. $P \Rightarrow Q$ and $Q \Rightarrow P$ are generally *not* the same.
$P \Leftrightarrow Q$ means both $P \Rightarrow Q$ and $Q \Rightarrow P$. $P \Leftrightarrow Q$ and $Q \Leftrightarrow P$ *are* the same.
P iff Q means $P \Leftrightarrow Q$, and means P and Q are saying exactly the same thing.

$\mathbb{R}$ is the set of real numbers.
$\mathbb{C}$ is the set of complex numbers (the complex plane).
$x = \mathrm{Re}(u)$ is the real part of the complex number $u = x + iy, \quad i = \sqrt{-1}$.
$y = \mathrm{Im}(u)$ is the imaginary part of $u = x + iy$. y is a real number.

$i = 0\!:\!n$ means $i = 0, 1, 2, \ldots, n$
$[a, b]$ is the interval (the set) of all $x \in \mathbb{R}$ such that $a \leq x \leq b$.
(a, b) is the interval (the set) of all $x \in \mathbb{R}$ such that $a < x < b$.
$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ is the set of all ordered pairs (x, y) with $x, y \in \mathbb{R}$ (the xy-plane).
(x, y) might be a point in $\mathbb{R}^2$ or an interval in $\mathbb{R}$; we will know by context.
$[a, b] \times [c, d]$ is the set of all points (x, y) such that $a \leq x \leq b$ and $c \leq y \leq d$.
$\mathbb{C}^2 = \mathbb{C} \times \mathbb{C}$ is the set of all ordered pairs (u, v) with $u, v \in \mathbb{C}$.

$f : D \to R$ is a function f with domain D and range R.
f' is the derivative function of the function f (aka $\frac{df}{dx}$ or Df).
f' is used for real (or complex) functions of a single real (or complex) variable.
$f'(x)$ is the derivative of the function f evaluated at the point x (aka $\frac{df}{dx}(x)$ or $(Df)(x)$).
$f^{(k)}$ is the k^{th} derivative of the function f: $\frac{d^k f}{dx^k}$ or $D^k f$ (so $f'' = f^{(2)}$, $f''' = f^{(3)}$,...)
$f \in C^0(D)$ means f is a continuous function on the domain D.
$f \in C^1(D)$ means f' is a continuous function on the domain D.
$f \in C^k(D)$ means $f^{(k)}$ is a continuous function on the domain D.
$f \in C^\infty(C)$ means f can be differentiated infinitely many times.
$C^k(D)$ is the set of all functions having k continuous derivatives on the domain D.
Functions $C^k(D)$ are *smoother* than functions only in $C^{(k-1)}(D)$:
...smoother as we go left-to-right: $C^0(D) \supset C^1(D) \supset C^2(D) \supset \cdots \supset C^\infty(D)$.

$\int f(x)\, dx$ is the set of real functions whos derivative is f.
$\int_a^b f(x)\, dx$ is the definite integral ('area under curve') of f from a to b. It's a number.
$\int_\Omega f\, d\Omega$ is the 'volume' under the 'surface' over Ω. It's a number.
2-D example: $\int_c^d \int_a^b f(x, y)\, dx\, dy = \int_\Omega f\, d\Omega$, with $\Omega = [a, b] \times [c, d]$ and $d\Omega = dx\, dy$.

p or P is a polynomial (in indeterminate x) if it can be written as $p = \sum_{i=0}^n a_i x^i$.
$p(x)$ or $P(x)$ is the value of p when evaluated at the number x.
$\deg(p)$ is the *degree* of p (the largest subscript i for which $a_i \neq 0$). It is an integer ≥ 0.

$\mathrm{fl}(x)$ is the *floating point representation* of x (the float nearest x).
$\boldsymbol{\mu}$ is the *unit round-off* (half the distance between 1 and the next float).
$f = \mathrm{O}(h^k)$ means $|f(h)| \leq C|h|^k, \ \forall |h| \leq \delta$, for some positive constants C and δ.