Systems of IVP's

$$y'(t) = f(t, y), \quad t \in [a, b], \quad y(a) = \alpha \qquad \text{(IVP)}$$

If we interpret y, α and f as

$$y : [a, b] \to \mathbb{R}^n, \quad \alpha \in \mathbb{R}^n, \quad \text{and} \quad f : [a, b] \times \mathbb{R}^n \to \mathbb{R}^n,$$

then (IVP) represents a system of IVP's. Remarkably, all of the methods that we have investigated continue to work in this more general setting. If the computer language supports vector operations, then the code can be identical; if not, then statements like

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w(j+1) = w(j) + h*ϕ(t,w(j))
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need to be replaced by a loop

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for i=1:n,  w(i,j+1) = w(i,j) + h*Φ(t,w(i,j));  end.
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For adaptive methods, quantities like $|w_{j+1} - w^*_{j+1}|$ will need to be replaced by $\|w_{j+1} - w^*_{j+1}\|$ for some vector norm $\| \cdot \|$. This means that adaptive methods will take a timestep that is safe for the *all* the components of the solution vector, and thus the most oscillatory component determines the timestep.

The simplest system of IVP's is linear, homogeneous and time-invariant, like

$$y'(t) = Ay(t), \quad t \in [a, b], \quad y(a) = \alpha,$$

where $A \in \mathbb{R}^{n \times n}$ is a constant matrix. We can write down the analytic solution to this problem in terms of eigenvalues and eigenvectors of A (or the matrix e^{At}), but for nearly all other systems, we will need our numerical methods.

Many systems of IVP's come from higher order scalar equations and partial differential equations. We will talk here about higher order scalar equations. Consider the very general IVP

$$y^{(m)}(t) = f(t, y, y', y'', \ldots, y^{(m-1)}), \quad t \in [a, b], \quad y^{(k)}(a) = \alpha_k, \quad k = 0, 1, \ldots, m-1.$$

By defining the new variables

$$u_1(t) = y(t), u_2(t) = y'(t), \ldots, u_m(t) = y^{(m-1)}(t)$$

the m^{th} order system above becomes the $m \times m$ first order system

$$\mathbf{u}'(t) = F(t, \mathbf{u}), \quad t \in [a, b], \quad \mathbf{u}(a) = \alpha, \text{ where}$$

$$F(t, \mathbf{u}) = \begin{pmatrix} u_2(t) \\ u_3(t) \\ \vdots \\ u_m(t) \\ f(t, \mathbf{u}) \end{pmatrix}.$$

The ability of our methods to "solve" systems of first order IVP's combined with the ability to transform higher order IVP's to first order systems means the methods we have been studying are very general IVP solvers.