(20) 1. General

(a) State the initial value problem that we have been calling (IVP).

(b) Describe the difference between the actual solution to (IVP) and the output of our IVP methods.

(c) Give a short description, in words, of a well-posed IVP.

(d) Describe the behavior of a stiff IVP.

(40) 2. Single step methods

(a) Approximate the solution to $y' = y^2 - 2t, \quad 0 \leq t \leq \frac{1}{2}, \quad y(0) = 2,$ using Euler's method with $h = 0.5$.

(b) Describe the theorem on the (total) error in Euler's method. Comment on its significance.

(c) Define local truncation error for single-step methods.

(d) Define the Taylor method of order n and explain why it is not a general purpose method.

(e) Are Runge-Kutta methods general purpose? Why or why not?

(40) 3. Multistep methods

(a) Give a coarse derivation of the Adams type multistep methods.

(b) Describe what an implicit multistep method is.

(c) Describe what a predictor-corrector method is.

(d) The Adams-Bashforth 2 step method has formula

$$w_{i+1} = w_i + \frac{h}{2}(3f(t_i, w_i) - f(t_{i-1}, w_{i-1})).$$

Use this method to approximate the solution to
$y' = t - y, \quad 0 \le t \le 1, \quad y(0) = 2$, using $h = 0.5$ and $w_1 = 1$.
Circle your approximation to $y(1)$.