Test 3 Math 4363 – Numerical Analysis Fall nnnn

(5) 1. State a theorem regarding the existence and uniqueness of solutions for (IVP).

(5) 2. Give a short description, in words, of a well-posed IVP.

(5) 3. Let $f(0) = 1$, $f(.25) = 2$, $f(.5) = 4$, $f(.75) = 5$, $f(1) = 5$. Approximate $f'(.5)$ using any 3-pt. formula.

(10) 4. Write down any difference formula for $f'(x)$ (with its error term), and using that formula describe why numerical differentiation is difficult.

(30) 5. Numerical Integration

(a) Approximate $\int_0^2 3x^3 \, dx$ using Simpson's rule. You do not need to simplify your result.

(b) Describe the adaptive quadrature method in as much detail as you can.

(c) What is the error term for Monte Carlo integration on n samples? When is Monte-Carlo integration most often used? Why?

(45) 6. IVP methods

(a) Approximate the solution to $y' = t^2 - 2y, \quad 0 \leq t \leq 1, \quad y(0) = 1$, using Euler's method with $h = 0.5$. Circle your approximation to $y(1)$.

(b) Describe the theorem on the (total) error in Euler's method. Comment on its significance.

(c) Define local truncation error for single-step methods.

(d) Define the Taylor method of order n and explain why it is not a general purpose method.

(e) Are Runge-Kutta methods general purpose? Why or why not?

(f) Describe the difference between the actual solution to (IVP) and the output of our IVP methods.