(48) 1. Numerical Integration

(a) Approximate $\int_2^3 2x^5 \, dx$ using Simpson's rule. You do not need to simplify your result.

(b) Derive the error estimator for the (Simpson's) adaptive quadrature method.

(c) Starting from the trapezoid method, derive the composite trapezoid method (you do not need to simplify the error term).

(d) Describe what Monte-Carlo integration is.

(52) 2. IVP methods

(5) (a) State the general form of the initial value problem that we have been trying to solve.

(12) (b) Approximate $y(1/2)$, where y is the solution to $y' = 2t - y,\ t \in [0, 1/2],\ y(1/2) = 1$, using Euler's method with $h = 0.25$.

(12) (c) A single-step IVP solver has the general form

$$w_{k+1} = w_k + h\phi(t_k, w_k).$$

Please give short answers to the following:

i. What is w_k?

ii. What is h?

iii. What is the relation between $a,\ b,\ h$, and N.

iv. If $w_k = y(t_k)$, what value of $\phi(t_k, w_k)$ would give $w_{k+1} = y(t_{k+1})$? Interpret this.

(5) (d) Define local truncation error for a single-step method.

(9) (e) Show that the Talyor method of order n has local truncation error $O(h^n)$.

(9) (f) Write down the Runge-Kutta iteration given by the following description: Using the Euler slope, s_1, sample the slope field at time $t_k + h/3$. Call this slope s_2. Using the the slope $s_3 = (s_1 + s_2)/2$, sample the slope field at time $t_k + 2h/3$. Call this slope s_4. Now update w using $\phi = (s_1 + s_2 + s_3 + 2s_4)/5$. Put your final answer right here:

$w_{k+1} =$ ____