Name: _____

(30) 1. Numerical Differentiation

(a) Derive any numerical differentiation formula you like (including the error term).

(b) Let $f(x) = 3x^3$. Approximate $f'(0.2)$ using the 3-point centered difference formula with $h = 0.1$.

(c) Argue the importance of high order differentiation formulas in the presence of rounding errors.

(30) 2. Numerical Integration

(a) Approximate $\int_1^2 x^4 \, dx$ using Simpson's rule. You do not need to simplify your result.

(b) Describe the adaptive quadrature method in as much detail as you can.

(c) Is numerical quadrature unstable (like differentiation), or stable? Why? Hint: Choose a simple method (e.g. composite trapezoid) and describe what happens to truncation and rounding errors as $h \to 0$.

(20) 3. IVP Theory

(a) State the general form of the initial value problem that we have been trying to solve.

(b) State a theorem regarding the existence and uniqueness of solutions for this problem.

(c) Define Lipschitz continuity.

(20) 4. IVP methods

(a) Approximate $y(2)$, where y is the solution to $y' = t - 2y,\ t \in [1, 2],\ y(1) = 2$, using Euler's method with $h = 0.5$.

(b) Write down $T^{(n)}$ and describe the difficulties associated with the implentation of Taylor methods for $n > 1$.