Name: ______________________________

(25) 1. Let $P_L(x)$ be the Lagrange polynomial for f on $x_0 < x_1 < \ldots < x_n$.

(a) Provided f is smooth enough, what is the error term for approximating $f(x)$ by $P_L(x)$ for $x \in [x_0, x_n]$? What, if anything can be said about ξ?

(b) If P_H is the Hermite interpolator for f on these nodes, then what is the (maximum possible) degree for P_H?

(c) If f is a polynomial of degree $n-2$, then what is P_L (in terms of f)?

(10) 2. Let $(x_0, y_0), (x_1, y_1), \ldots (x_n, y_n)$ be points in $\mathbb{R}^2$ with $x_i \neq x_j$ for $i \neq j$. Describe cubic spline interpolant for this data (what is it and what properties does it have?).

(15) 3. General Numerical Differentiation

(a) Write down the 2-point forward difference formula with its truncation error term.

(b) Discuss the difference between truncation error and rounding error.

(c) Explain why you cannot, in general, expect to get high accuracy using this formula with floating point arithmetic.

(21) 4. Numerical Integration

(a) Approximate $\displaystyle\int_0^4 x^5\,dx$ using Simpson's rule.

(b) Now approximate the same integral using a composite Simpson's rule with $n = 4$.

(c) The Newton-Cotes quadrature rules can be derived from Lagrange interpolators, and have the form $\int_a^b f(x)\,dx \approx \sum_{j=0}^{n} c_j f(x_j)$; how are the c_j defined?

(14) 5. Let $f(x) = x^3 + 2x$.

(a) Approximate $f'(0)$ using the 3 point forward difference formula

$$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)] + h^2 f'''(\xi)/3$$

with $h = 0.5$.

(b) Compare the error in your approximation to the predicted truncation error.

(15) 6. Let $f(0) = 3,\ f(1) = 2,\ f(3) = 3,\ f'(0) = 5,$ and $f'(3) = -1$. Let S be a clamped cubic spline for this data, with $S_0(x) = a_0 + b_0 x + c_0 x^2 + d_0 x^3$ and $S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3$.

(a) $a_0 =$

(b) $a_1 =$

(c) $b_0 =$

(d) $c_0 + d_0 =$

(e) $2c_0 + 6d_0 - 2c_1 =$