(15) 1. Let P be a polynomial of degree n in standard form with real coefficients.

(a) Describe how deflation might be used to compute all of the roots of P.

(b) How many multiplications are required to evaluate P at a real number x_0? Show how.

(c) Describe Mueller's method. (Hint: if you like you may explain it graphically).

(5) 2. State the Weierstrass approximation theorem.

(40) 3. Let $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ be real ordered pairs with $x_i \neq x_j$ for $i \neq j$.

(a) Give the general definition of the osculating polynomial for this data. Show how Lagrange Interpolation, Hermite interpolation, and the Taylor Polynomial are special cases.

(b) Write down the error term for the Lagrange interpolator for this data, assuming that $y_i = f(x_i)$ for a sufficiently smooth function f.

(c) Write down the conditions which define the cubic spline interpolator for this data. Describe at least 2 of 3 popular boundary conditions.

(d) Let P_L be the Lagrange interpolator for this data. If a polynomial of degree $n-2$ also interpolates this data, what can be said about the degree of P_L?

(20) 4. Consider the following data: $f(0) = 1$, $f(1) = 1$ and $f(3) = 2$.

(a) Approximate $f(2)$ using a degree 2 interpolator for this data.

(b) Write down the normal equations for the linear least squares approximation for this data.

(5) 5. What is an ill-conditioned problem?

(5) 6. What is the absolute condition number for the root finding problem "find x^* so that $f(x^*) = 0$"? Describe the conditions for which the root finding problem is ill-conditioned.

(10) 7. Let $\phi_1(x), \phi_2(x), \ldots, \phi_m(x)$ be a set of functions linearly independent over some interval containing $x_1, x_2, \ldots, x_n$. Describe the least squares problem of approximating the data $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ by a linear combination of the ϕ_j (what are we trying to find, and what does it satisfy?).