Name: ____________________

(30) 1. Let $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ be real ordered pairs with $x_i \neq x_j$ for $i \neq j$, and $x_i \in [a, b]$.

(a) Give the formulae for the Lagrange basis functions and the Lagrange interpolating polynomial P for this data.

(b) Give the truncation error term for the Lagrange interpolator for this data, assuming that $y_i = f(x_i)$ for a sufficiently smooth function f. Include as much information as possible about any parameters given in your answer.

(c) Let $P_L(x)$ be the Lagrange interpolator from part (a) above. If a polynomial of degree n-1 also interpolates this data, what can be said about the degree of P_L?

(d) Let $P_H(x)$ be the Hermite interpolator for these knots, where also y_i' is given at each of the nodes. What is the (maximum possible) degree of P_H?

(15) 2. Let $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ be real ordered pairs with $x_0 < x_1 < \cdots < x_n$.

(a) Describe the cubic spline interpolator, $S(x)$, for this data and define the conditions it must satisfy.

(b) Are boundary conditions necessary? Explain.

(c) At which points in (x_0, x_n) might $S'''(x)$ be undefined?

(8) 3. Let $f(1) = 0$, $f(3) = 3$ and $f(4) = 2$. Approximate $f(2)$ using a degree 2 Lagrange interpolator.

(8) 4. Let $f(0.5) = 2$, $f(1) = 1$, and $f(1.5) = 1$. Approximate $f'(1)$ using the 3-pt. centered difference formula.

(9) 5. Write down any difference formula for $f'(x)$ (with its error term), and using that formula describe why numerical differentiation is difficult.

(30) 6. Numerical Integration

(a) Approximate $\int_0^1 x^4 \, dx$ using Simpson's rule. You do not need to simplify your result.

(b) Describe an adaptive quadrature method in as much detail as you can.

(c) Briefly describe *composite* Newton-Cotes quadrature and explain why this *can* achieve high accuracy.

(d) When is Monte-Carlo integration most often used? Why?