Math 4363 – Numerical Analysis | Test 2 | Fall

Name: ________________________

(20) 1. Let $S(x) = \begin{cases} 1 + 2x^2 + x^3, & x \in [1,2) \\ 25 - 36x + bx^2 - 2x^3, & x \in [2,3] \end{cases}$ be a cubic spline function.

(a) Find b.

(b) Find $S(1.5)$.

(c) Is S a *natural spline* on this interval? Explain.

(10) 2. Suppose we have a function f that we would like to approximate. We have data for f at 5 nodes. At node j we know all of the derivatives of f through the m_j^{th}, where $m_0 = 2,\ m_1 = 1,\ m_2 = 3,\ m_3 = 2$, and $m_4 = 2$. We can expect to interpolate f and its derivatives at these points with an osculating polynomial of degree less than or equal to d. What is d?

(10) 3. Suppose f is a sufficiently smooth function and $x_0, x_1, \ldots, x_n$ are distinct.

(a) Define an osculating polynomial for f on the nodes.

(b) Define the Lagrange interpolating poly. in terms of the osculating polynomial.

(c) Define the Hermite interpolating polynomial in terms of the osculating polynomial.

(15) 4. Let $P_L(x)$ be the Lagrange polynomial for f on $x_0, \ldots, x_n \in [a,b]$.

(a) What is the error in approximating $f(x)$ by $P_L(x)$ for $x \in [a,b]$?

(b) What can be said about a polynomial, P, which has the same degree as P_L, and which also interpolates f on $x_0, \ldots, x_n$?

(c) Let $L_{n,k}(x)$ be a Lagrange basis function for this data. What is $L_{n,k}(x_j)$?

(20) 5. Let $x_0 = 1,\ x_1 = 2$ and $x_2 = 3$ and $f(x_0) = 1$, $f(x_1) = 2$ and $f(x_2) = 4$.

(a) Set up the normal equations for the least squares line for this data.

(b) Approximate $f(1.5)$ with a degree 2 Lagrange polynomial.

(10) 6. State the Weierstrass approximation theorem.

(15) 7. Let $(x_0, y_0),\ (x_1, y_0),\ \ldots,\ (x_n, y_n)$ be points in the xy-plane, with the x_j pairwise distinct. Discuss the similarities and differences between the interpolating polynomial, the cublic spline, and the least squares approximating polynomial for these data.