1. Let $S(x) = \begin{cases} 1 + 2x^2 + x^3, & x \in [1, 2) \\ 25 - 36x + bx^2 - 2x^3, & x \in [2, 3] \end{cases}$ be a cubic spline function.

(a) Find $b$.
(b) Find $S(1.5)$.
(c) Is $S$ a natural spline on this interval? Explain.

2. Suppose we have a function $f$ that we would like to approximate. We have data for $f$ at 5 nodes. At node $j$ we know all of the derivatives of $f$ through the $m_j^{th}$, where $m_0 = 2$, $m_1 = 1$, $m_2 = 3$, $m_3 = 2$, and $m_4 = 2$. We can expect to interpolate $f$ and its derivatives at these points with an osculating polynomial of degree less than or equal to $d$. What is $d$?

3. Suppose $f$ is a sufficiently smooth function and $x_0, x_1, \ldots, x_n$ are distinct.

(a) Define an osculating polynomial for $f$ on the nodes.
(b) Define the Lagrange interpolating poly. in terms of the osculating polynomial.
(c) Define the Hermite interpolating polynomial in terms of the osculating polynomial.

4. Let $P_L(x)$ be the Lagrange polynomial for $f$ on $x_0, \ldots, x_n \in [a, b]$.

(a) What is the error in approximating $f(x)$ by $P_L(x)$ for $x \in [a, b]$?
(b) What can be said about a polynomial, $P$, which has the same degree as $P_L$, and which also interpolates $f$ on $x_0, \ldots, x_n$?
(c) Let $L_{n,k}(x)$ be a Lagrange basis function for this data. What is $L_{n,k}(x_j)$?

5. Let $x_0 = 1$, $x_1 = 2$ and $x_2 = 3$ and $f(x_0) = 1$, $f(x_1) = 2$ and $f(x_2) = 4$.

(a) Set up the normal equations for the least squares line for this data.
(b) Approximate $f(1.5)$ with a degree 2 Lagrange polynomial.

6. State the Weierstrass approximation theorem.

7. Let $(x_0, y_0), (x_1, y_0), \ldots, (x_n, y_n)$ be points in the $xy$-plane, with the $x_j$ pairwise distinct. Discuss the similarities and differences between the interpolating polynomial, the cubic spline, and the least squares approximating polynomial for these data.