(20) 1. Let $S(x) = \begin{cases} 1 + 2x + x^2 + x^3, & x \in [0, 1) \\ 5 + b(x-1) + 4(x-1)^2 + (x-1)^3, & x \in [1, 2] \end{cases}$ be a clamped cubic spline for a function f with $f'(0) = 1$.

(a) Find b.

(b) Find $S(0.5)$.

(c) Find $f'(2)$.

(d) Can we find $f'(1)$ from this spline? Explain.

(15) 2. Discuss the differenece between truncation error and rounding error. Then explain, with mathematical detail when you can, why numerical differentiation is unstable ("hard").

(15) 3. Answer the following:

(a) Suppose f is a sufficiently smooth function and $x_0, x_1, \ldots, x_n$ are distinct. Define an osculating polynomial for f on the nodes.

(b) Define the Lagrange interpolating polynomial in terms of the osculating polynomial.

(c) Define the Taylor polynomial in terms of the osculating polynomial.

(d) Define the Hermite interpolating polynomial in terms of the osculating polynomial.

(10) 4. Let $P_L(x)$ be the Lagrange polynomial for f on $x_0, \ldots, x_n \in [a, b]$.

(a) What is the error in approximating $f(x)$ by $P_L(x)$ for $x \in [a, b]$?

(b) What can be said about another polynomial, P, which is different than P_L, but which also interpolates f on $x_0, \ldots, x_n \in [a, b]$?

(20) 5. Let $x_0 = 1,\ x_1 = 2$ and $x_2 = 4$ and $f(x_0) = 1$, $f(x_1) = 3$ and $f(x_2) = 5$.

(a) Set up the normal equations for the least squares line for this data. (You don't need to solve, but write the equations with this data).

(b) Approximate $f(3)$ with a degree 2 Lagrange polynomial.

(10) 6. Let $f(x) = 2x^3$.

(a) Approximate $f'(0.2)$ using the 3-point centered difference formula with $h = 0.1$.

(b) Give the 2-point forward error formula (including error term).

(10) 7. Let S be a cubic spline interpolant defined on the nodes $x_0 = 1,\ x_1 = 2,\ x_2 = 3$.

(a) Make a sketch of a typical $S'''(x)$.

(b) Make a sketch of a typical $S''(x)$.