(40) 1. Let $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ be real ordered pairs with $x_i \neq x_j$ for $i \neq j$.

(a) Give the general definition of the osculating polynomial for this data. Show how Lagrange Interpolation, Hermite interpolation, and the Taylor Polynomial are special cases.

(b) Write down the error term for the Lagrange interpolator for this data, assuming that $y_i = f(x_i)$ for a sufficiently smooth function f.

(c) Let P_L be the Lagrange interpolator for this data. If a polynomial of degree $n - 2$ also interpolates this data, what can be said about the degree of P_L?

(d) Write down the conditions which define the cubic spline interpolator for this data. Describe at least 2 of 3 popular boundary conditions.

(10) 2. Let $f(0) = 1$, $f(1) = 1$ and $f(3) = 2$. Approximate $f(2)$ using a degree 2 interpolator.

(10) 3. Let $f(0) = 1$, $f(.25) = 2$, $f(.5) = 2$, $f(.75) = 3$, $f(1) = 4$. Approximate $f'(.5)$ using any 3-pt. formula.

(10) 4. Write down any difference formula for $f'(x)$ (with its error term), and using that formula describe why numerical differentiation is difficult.

(30) 5. Numerical Integration

(a) Approximate $\int_0^2 5x^3 \, dx$ using Simpson's rule. You do not need to simplify your result.

(b) Describe the adaptive quadrature method in as much detail as you can.

(c) Briefly describe Monte Carlo integration.

(d) When is Monte-Carlo integration most often used? Why?