Name: _____

(6)     1. Computing the zeros of $x^2 + 10x - \frac{1}{20}$, a programmer writes

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

which root will have a smaller relative error and why?

(4)     2. What is the fundamental axiom of floating point arithmetic?

(4)     3. Argue that secant is better than bisection.

(4)     4. Argue that bisection is better than secant.

(18)     5. Let $f(x) = x^2 + 2x - 4$. We're looking for a zero of $f$.

(a) Use the bisection method with $a = 0$ and $b = 2$ to find an interval of length strictly less than 1 which brackets a zero of $f$.

(b) Use one iteration of Newton's method to improve the guess $x_0 = 1$.

(c) With $x_0 = 2$ and $x_1 = 1$, use one iteration of the secant method to find $x_2$.

(12)     6. Root finding

(a) Give a definition for the *order of convergence* of a sequence.

(b) Let $p$ be a polynomial of degree $n$, and suppose you have a method that can approximate 1 root of any polynomial. Carefully describe how *deflation* is used to approximate all of the roots of $p$.

(5)   7. State the Taylor polynomial theorem (give the hypotheses and the formulas for $P_n$ and $R_n$).

(25)  8. Finite precision arithmetic.

   (a) Let $a = 0.032994$ and $b = 4.074$. Compute the 3 (decimal) digit rounding representations of $a$ and $b$; call them $\bar{a}$ and $\bar{b}$, respectively.

      i. $\bar{a} =$

      ii. $\bar{b} =$

      iii. What is the floating point result of $\bar{a} + \bar{b}$?

   (b) What do we mean by underflow?

   (c) What is cancellation?

(10)      9. Let $f(x) = \sqrt{1-x}$.

   (a) Compute $P_2(x)$, the degree 2 Taylor polynomial for $f$ at $x_0 = 0$.

   (b) Use $P_2$ to approximate $f(0.5)$.

(6)     10. How many multiplications are required to evaluate a real polynomial of degree $n$ at a real number? Explain.

(6)     11. Let $x$, $y$, and $xy$ are real numbers (not necessarily floats) in the range of our floating point system. Find an upper bound for the relative error in the *computed* product of $x$ and $y$.