Fall nnnn

Test 1

- (25) 1. Finite precision arithmetic.
 - (a) Let a = 0.0079346 and b = 42.0963. Compute the 2 (decimal) digit rounding representations of a and b, call them \bar{a} and \bar{b} respectively.
 - i. $\bar{a} =$
 - ii. $\bar{b} =$
 - iii. What is the machine precision, μ , for this arithmetic?
 - (b) What do we mean by underflow?

(c) What is digit cancellation?

(15) 2. Let x and y be real numbers such that none of x, y nor xy underflow nor overflow. Derive an upper bound on the relative error in computing the floating point product of x and y.

- (30) 3. Let $f(x) = x^2 6$. We're looking for a zero of f.
 - (a) Use the bisection method with a = 2 and b = 3 to find an interval of length strictly less than 1/4 which brackets a zero of f.

(b) Use one iteration of Newton's method to improve the guess $x_0 = 3$.

(c) Starting with $x_0 = 3$ and $x_1 = 2.5$, use one iteration of the secant method to find x_2 .

- (d) Define the order of convergence of a sequence.
- (e) What is the order of convergence for bisection and the secant methods?
- (f) Briefly discuss the debate about the order of convergence for Newton's method.

(30) 4. Let $f(x) = e^{2x} \cos(x)$.

(a) State the Taylor polynomial theorem in its general form.

(b) Compute $P_2(x)$, the degree 2 Taylor polynomial for f at $x_0 = 0$.

(c) Use P_2 to approximate f(0.25).

(d) What is the remainder term associated with P_2 at x = 0.25?