Name: _____

(8) 1. Suppose $0 < \epsilon < a$. Let $b = a - \sqrt{a^2 - \epsilon}$. To compute b, which method below would you suggest and why?

(a) $c = \sqrt{a^2 - \epsilon}$; $b = a - c$;

(b) $c = \sqrt{a^2 - \epsilon}$; $b = \epsilon/(a + c)$;

(6) 2. Describe the set of floating point numbers using the terms *underflow*, *overflow* and *machine epsilon*.

(4) 3. For small h, $1 + 3h^2 + 5h^4 + 7h^5 \approx 1 + 3h^2$. Write this using the "big O" notation.

(3) 4. How many different numbers can be represented by 64 bits?

(24) 5. Let $f(x) = 9x^2 - 15x + 4$.

(a) Use the bisection method with $a = 0$ and $b = 1$ to find an interval of length strictly less than 1 which brackets a zero of f.

(b) Use one iteration of Newton's method to improve the guess $x_0 = 0$.

(c) With $x_0 = 0$ and $x_1 = 1$, use one iteration of the secant method to find x_2.

(6) 6. What is meant by *order of convergence* of a sequence $\{p_n\} \to p$?

(21) 7. Bisection, Newton's and secant methods.

(a) Discuss the *efficiency* of these methods (you may assume convergence).

(b) Compare and contrast the *generality* of the methods (more restrictions on inputs means less general method).

(c) Compare and contrast the *robustness* of the methods (do they converge, do they provide error estimates or error bounds, etc.).

(8) 8. Finite precision arithmetic.

9. Let $a = 0.00042854$ and $b = 4482.049$. Using 3 (decimal) digit floating point arithmetic, find a and b; call them $\bar{a}$ and $\bar{b}$, respectively.

   (a) $\bar{a} =$

   (b) $\bar{b} =$

(6) 10. What do we mean by *swamping* in floating point arithmetic?

(8) 11. Suppose you have a method, say M, to compute an approximate root of any polynomial. Describe a method to compute *all* n of the roots of a polynomial, say $p(x)$, of degree n.

(6) 12. How many multiplications are required to evaluate a real polynomial p of degree n at a real number s? Explain (you may use an example if you like).