Name: _____

(30) 1. Let $f(x) = x^2 + 2x - 3$. We're looking for a zero of f.

(a) Use the bisection method with $a = 0$ and $b = 6$ to find an interval of length strictly less than 3 which brackets a zero of f.

(b) Use one iteration of Newton's method to improve the guess $p_0 = 2$ (that is, find p_1).

(c) With $p_0 = 3$ and $p_1 = 2$, use one iteration of the secant method to find p_2.

(d) Compare and contrast properties related to performance and generality of bisection, Newton's method and the secant method.

(5) 2. Describe the stabilized deflation method for finding *all* of the roots of a polynomial of degree n.

(25) 3. Finite precision floating point arithmetic.

(a) Let $a = 0.0047927$ and $b = 240.56$. Compute the 3 decimal-digit rounding representations of a and b, call them $\bar{a}$ and $\bar{b}$ respectively.

i. $\bar{a} =$

ii. $\bar{b} =$

(b) Describe the machine epsilon in terms of the spacing between neighboring floats.

(c) What do we mean by *overflow*?

(d) What is *digit cancellation*?

(8) 4. Let $f(x) = \frac{1}{1-x}$.

(a) Compute $P_1(x)$, the degree 1 Taylor polynomial for f at $x_0 = 0$.

(b) Use P_1 to approximate $f(0.1)$.

(8) 5. How many multiplications are required to evaluate an arbitrary real polynomial of degree n at a real number? Explain.

(8) 6. Show that if x, y and xy are real numbers in the range of our floating point system, then

$$\frac{|xy - \text{fl}(xy)|}{|xy|} \leq 3\mu + \text{O}(\mu^2)$$

.

(10) 7. Conditioning

(a) What does *illconditioning* mean in a mathematical problem?

(b) What is the absolute condition number for the problem "find x^* so that $f(x^*) = 0$"?

(6) 8. State the fundamental axiom of floating point arithmetic.