Name: _____

(30) 1. Let $f(x) = x^2 + x - 2$. We're looking for a zero of f.

(a) Use the bisection method with $a = -6$ and $b = 0$ to find an interval of length strictly less than 3 which brackets a zero of f.

(b) Use one iteration of Newton's method to improve the guess $x_0 = -4$.

(c) With $x_0 = -4$ and $x_1 = -3$, use one iteration of the secant method to find x_2.

(d) Compare and contrast bisection, Newton's method and the secant method. Include in your discussion properties related to performance and generality.

(5) 2. Describe a popular method for finding *all* of the roots of a polynomial.

(25) 3. Finite precision arithmetic.

(a) Let $a = 0.00047964$ and $b = 24.046$. Compute the 3 (decimal) digit rounding representations of a and b, call them $\bar{a}$ and $\bar{b}$ respectively.

i. $\bar{a} =$

ii. $\bar{b} =$

i. What do we mean by *swamping*?

ii. What do we mean by *overflow*?

iii. What is *digit cancellation*?

(8) 4. Let $f(x) = \frac{1}{1+x}$.

(a) Compute $P_1(x)$, the degree 1 Taylor polynomial for f at $x_0 = 0$.

(b) Use P_1 to approximate $f(0.1)$.

(c) Use the remainder term to give an upper bound on the error.

(6) 5. How many multiplications are required to evaluate a real polynomial of degree n at a real number? Explain.

(6) 6. Show that if x, y and xy are real numbers in the range of our floating point system, then

$$\frac{|xy - \mathrm{fl}(xy)|}{|xy|} \leq 3\mu + \mathrm{O}(\mu^2)$$

.

(15) 7. Conditioning

(a) What does *illconditioning* mean in a computational problem?

(b) What is the absolute condition number for the problem "find x so that $f(x) = 0$"?

(c) Define *backward stability* in your own words.

(5) 8. State the floating point representation theorem?