Name: ____________________________

(32) 1. Let $f(x) = x^2 - 5x + 4$. We're looking for a zero of f.

(a) Use the bisection method with $a = 3$ and $b = 11$ to find an interval of length strictly less than 3 which brackets a zero of f.

(b) Use one iteration of Newton's method to improve the guess $p_0 = 3$ (that is, find p_1).

(c) With $p_0 = 3$ and $p_1 = 5$, use one iteration of the secant method to find p_2.

(d) What is the order of convergence, per function evaluation (α_f), of bisection, Newton's, and the secant methods, respectively?

(5) 2. Suppose you have used the secant method or Newton's method to generate the approximations x_0, x_1 and x_2 to a zero of a function f. Describe how to generate an improved estimate, say x_3, using Müller's method.

(10) 3. Let $f(x) = \frac{1}{1+x}$.

(a) Compute $P_1(x)$, the degree 1 Taylor polynomial for f at $x_0 = 0$.

(b) Use P_1 to approximate $f(0.1)$.

(33) 4. Finite precision floating point arithmetic.

(a) Let $a = 0.0047927$ and $b = 199.6477$. Compute the 3 decimal-digit (rounding) representations of a and b, call them $\bar{a}$ and $\bar{b}$ respectively.

i. $\bar{a} =$

ii. $\bar{b} =$

(b) Suppose we have a floating point system with *minnfloat* $= m$, unit roundoff $= \mu$, and where underflow is set to 0.

i. Suppose x and y are floats and $\mathrm{fl}(x + y) = x$. Give an upper bound on $|y|$.

ii. Now suppose x can be any real number.

A. Describe the solution set of the equation $\mathrm{fl}(x) = 0$.

B. Describe the solution set of the equation $\mathrm{fl}(1 + x) = 1$.

(6) 5. How many multiplications are required to evaluate an arbitrary real polynomial of degree n at a real number? Explain.

(6) 6. Let p be a polynomial of degree n, and suppose you have a method which can compute 1 root of any polynomial. Carefully describe a stabilized (or corrected) delfation process for approximating all of the roots of p.

(8) 7. Conditioning

(a) What is the absolute condition number for the problem "find x^* so that $f(x^*) = 0$"?

(b) When computing the zeros of $p(x) = ax^2 + bx + c$, does $b^2 \approx 4ac$ indicate well-conditioned or ill-conditioned zeros? Explain.