(30) 1. Let $f(x) = x^2 + x - 2$. We're looking for a zero of f.

(a) Use the bisection method with $a = 0$ and $b = 3$ to find an interval of length strictly less than 2 which brackets a zero of f.

(b) Use one iteration of Newton's method to improve the guess $x_0 = 1.5$.

(c) With $x_0 = 0$ and $x_1 = 3$, use one iteration of the secant method to find x_2.

(d) What is meant by *order of convergence* and what is the order of convergence for bisection, Newton's and the secant methods?

(5) 2. State the Taylor polynomial theorem.

(25) 3. Finite precision arithmetic.

(a) Let $a = 0.0056967$ and $b = 31.043$. Compute the 4 (decimal) digit rounding representations of a and b, call them $\bar{a}$ and $\bar{b}$ respectively.

i. $\bar{a} =$

ii. $\bar{b} =$

iii. What is the machine precision, μ, for this (4 decimal digit, rounding) arithmetic?

(b) What do we mean by underflow?

(c) Carefully describe digit cancellation?

(8) 4. Let $f(x) = x + \sqrt{x}$.

(a) Compute $P_2(x)$, the degree 2 Taylor polynomial for f at $x_0 = 0$.

(b) Use P_2 to approximate $f(0.5)$.

(6) 5. How many multiplications are required to evaluate a real polynomial of degree n at a real number? Explain.

(6) 6. Show that if x, y and xy are real numbers in the range of our floating point system, then find an upper bound for

$$\frac{|xy - \text{fl}(xy)|}{|xy|}$$

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(15) 7. Conditioning

(a) What is a *well conditioned* problem?

(b) What is the absolute condition number for the problem "evaluate $f(x)$ at $x = x_0$"?

(c) Define backward stability in your own words.

(5) 8. What is the fundamental axiom of floating point arithmetic?