

- (30) 1. Let $f(x) = x^2 - 7$. We're looking for a zero of f .
- (a) Use the bisection method with $a = 2$ and $b = 3$ to find an interval of length strictly less than $1/2$ which brackets a zero of f .

 - (b) Use one iteration of Newton's method to improve the guess $x_0 = 3$.

 - (c) Starting with $x_0 = 3$ and $x_1 = 2.5$, use one iteration of the secant method to find x_2 .

 - (d) What is meant by *order of convergence* and what is the order of convergence for bisection, Newton's and the secant methods? Briefly discuss the debate about the order of convergence for Newton's method.

(5) 2. State the Taylor polynomial theorem.

(25) 3. Finite precision arithmetic.

(a) Let $a = 0.0037934$ and $b = 24.096$. Compute the 3 (decimal) digit rounding representations of a and b , call them \bar{a} and \bar{b} respectively.

i. $\bar{a} =$

ii. $\bar{b} =$

iii. What is the machine precision, μ , for this arithmetic?

(b) What do we mean by overflow?

(c) What is digit cancellation?

(12) 4. Let $f(x) = e^x \cos(x)$.

(a) Compute $P_2(x)$, the degree 2 Taylor polynomial for f at $x_0 = 0$.

(b) Use P_2 to approximate $f(0.5)$.

(4) 5. Explain how Horner's method is a method for dividing a polynomial by $(x - a)$, for some number a . Hint: you can write down the theorem we associated with Horner's method. How many multiplications are required to evaluate a real polynomial of degree n at a real number?

(4) 6. Show that $O(h^2) + O(h^3) = O(h^2)$.

(15) 7. Conditioning

(a) What does *illconditioning* mean in a computational problem?

(b) What is the absolute condition number for the problem “evaluate $f(x)$ at $x = x_0$ ”?

(c) When is the problem “find x so that $f(x) = 0$ ” illconditioned?

(5) 8. Does digit cancellation violate the fundamental axiom of floating point arithmetic (FAFA)? Explain.