Name: _____

(8)   1. Computing the zeros of $x^2 - 10x + \frac{1}{10}$, a programmer writes

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

which root will have a smaller relative error and why?

(5)   2. What is the fundamental axiom of floating point arithmetic?

(5)   3. Which of bisection, Newton and secant do you think is the most general purpose and why?

(18)   4. Let $f(x) = x^2 + 2x - 2$. We're looking for a zero of f.

(a) Use the bisection method with $a = 0$ and $b = 2$ to find an interval of length strictly less than 1 which brackets a zero of f.

(b) Use one iteration of Newton's method to improve the guess $x_0 = 1$.

(c) With $x_0 = 2$ and $x_1 = 1$, use one iteration of the secant method to find x_2.

(12)   5. Root finding

(a) What is meant by *order of convergence* of a sequence?

(b) Assuming it requires 1 day to compute $f(x_k)$, which of the three methods would you use and why?

(5)   6. State the Taylor polynomial theorem.

(25)   7. Finite precision arithmetic.

   (a) Let $a = 0.0032964$ and $b = 24.046$. Compute the 3 (decimal) digit rounding representations of a, b, and $a + b$; call them $\bar{a}$, $\bar{b}$ and $\bar{c}$ respectively.

      i. $\bar{a} =$

      ii. $\bar{b} =$

      iii. $\bar{c} =$

   (b) What do we mean by underflow?

   (c) What is swamping?

(10)   8. Let $f(x) = \sqrt{1 + 2x}$.

      (a) Compute $P_2(x)$, the degree 2 Taylor polynomial for f at $x_0 = 0$.

      (b) Use P_2 to approximate $f(0.5)$.

(6)   9. How many multiplications are required to evaluate a real polynomial of degree n at a real number? Explain.

(6)   10. If x, y and xy are real numbers in the range of our floating point system, then find an upper bound for the relative error
$$\frac{|xy - \text{fl}(xy)|}{|xy|}$$
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