(20) 1. Finite precision arithmetic.

(a) Let $a = 0.0094332$ and $b = 53.1961$. Compute the 2 (decimal) digit rounding representations of a and b, call them $\bar{a}$ and $\bar{b}$ respectively.

i. $\bar{a} =$

ii. $\bar{b} =$

(b) What do we mean by underflow?

(c) What is cancellation?

(10) 2. Let x and y be real numbers (not necessarily floats) such that none of x, y nor xy underflow nor overflow. Derive an upper bound on the relative error in computing the floating point product of x and y.

(30) 3. Let $f(x) = x^2 - 5$. We're looking for a zero of f.

(a) Use the bisection method with $a = 2$ and $b = 3$ to find an interval of length strictly less than $1/2$ which brackets a zero of f.

(b) Use one iteration of Newton's method to improve the guess $x_0 = 2$.

(c) Starting with $x_0 = 2$ and $x_1 = 2.5$, use one iteration of the secant method to find x_2.

(d) Define the *order of convergence* of a sequence.

(e) What is the order of convergence for bisection and the secant methods?

(10) 4. State the Taylor polynomial theorem in its general form.

(10) 5. When is the result of a computation considered *backward stable*?

(10) 6. When is a problem considered *ill-conditioned*?

(10) 7. Briefly describe a method for factoring polynomials of degree $n > 5$.