Math 4363 – Numerical Analysis Fall xxxx Test 1

Name: _____

(30) 1. Let $f(x) = x^3 - 5$. We're looking for a zero of f.

(a) Use the bisection method with $a = 1$ and $b = 3$ to find an interval of length strictly less than 1 which brackets a zero of f.

(b) Use one iteration of Newton's method to improve the guess $x_0 = 1$.

(c) Starting with $x_0 = 1$ and $x_1 = 1.5$, use one iteration of the secant method to find x_2.

(d) What is meant by *order of convergence* and what is the order of convergence for bisection, Newton's and the secant methods?

(5) 2. State the floating point representation theorem (FRT).

(25) 3. Let $a = 0.038734$ and $b = 42.089$.

(a) Compute the 3 (decimal) digit chopping representations of a and b, call them $\bar{a}$ and $\bar{b}$ respectively.

i. $\bar{a} =$

ii. $\bar{b} =$

(b) What is the unit roundoff, μ, for this arithmetic?

(c) What do we mean by overflow and underflow?

(d) What is digit cancellation?

(e) Give a bound for the relative difference between 2 neighboring positive floats.

(12) 4. Let $f(x) = x\sin(x)$.

(a) Compute $P_2(x)$, the second Taylor polynomial for f at $x_0 = 0$.

(b) Use P_2 to approximate $f(0.5)$.

(8) 5. Polynomial Roots

(a) Give a brief geometric description of Mueller's method. (A well annotated picture would suffice.)

(b) Describe how deflation can be used to compute all of the roots of a polynomial.

(15) 6. Conditioning

(a) What does *illconditioning* mean in a computational problem?

(b) What is the absolute condition number for the problem "evaluate $f(x)$ at $x = x_0$"?

(c) Suppose f has only one zero. What is the absolute condition number for the problem "solve $f(x) = 0$"?

(5) 7. State the fundamental axiom of floating point arithmetic (FAFA).